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Angular constraints in cold d-t fusion catalysed by negative muons

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Negative muons in matter can replace electrons and form atoms and molecules with the nuclei of matter. The muonic orbits of these muon flavoured atoms and molecules are 206 times smaller than their electronic counterparts because of the correspondingly larger muonic mass. In muonic molecular ions of Hydrogenic isotopes, the nuclei are therefore 206 times closer than in the electronic systems. This small spatial separation of the nuclei gives rise to a high probability of sub-barrier cold fusion.

In recent years the subject of cold fusion catalysed by negative muons has become a frontline research subject (Jones 1986, Bhatia and Drachman 1989), both for its rich fundamental physics content and for its application potential.

The problem of sticking (Hu 1986, Bogdonava *et al* 1986), whereby the muon forms a Coulomb bound state with the charged fusion products, is a critical bottleneck to efficient utilisation of muon catalysed cold fusion. Current experiments and theory therefore focus on evaluation of the sticking factor. Since the d-t system is blessed with a lot of fusion advantages, this combition of fusion fuel is usually studied most.

Existing theoretical descriptions of sticking have of late aroused some discontent (Hale *et al* 1988, Danos *et al* 1988). Recently calculation of the phase space corrections (Chatterjee 1988, 1989), applying correctly the different phase space constraints for the stuck and free fusion modes have yeilded a reduction of sticking compatible with experiments.

We demonstrate in this note that the conservation delta functions introduce angular constraints on the accessible phase space for the non-stuck fusion modes. These could be easily detectable in experiments of the type in progress to determine direct sticking (Paciotti *et al* 1988, Davis *et al* 1988).

For the stuck modes the phase space is trivial and is not studied in this work. However the phase space constraints obtained here for the non-stuck modes will be ultimately reflected in the branching ratio for sticking as this represents the ratio of the stuck to total final states.

Restricting the analysis to understanding the phase space constraints for the non-stuck fusion events for the d-t scenario, the final post fusion scence has three free particles, the alpha, the neutron and the muon. The entrance channel passes through the $(dt\mu)$ muomolecular bound state and the physics of reactions can be represented as

$$d+t+\mu \rightarrow (dt\mu) \begin{cases} \rightarrow \alpha+\mu+n & \text{[(a) final free muon]} \\ \rightarrow (\alpha\mu)+n & \text{[(b) stuck case]} \end{cases} \tag{1}$$

for the two exit channels, channel (1a) being the dominent one with (1b) providing 1% correction.

The dynamics of eq. (1a) involve the strong nuclear force effecting sub-barrier fusion of the nuclei in the presence of their electromagnetically connected partner the muon and the sharing of the available fusion energy between the three final partners of nondegenerate masses.

In the framework of the sudden approximation, generally used in muon catalysed fusion problems, the matrix element is given by the overlap of the initial and final wavefunctions. Thus,

$$M = G\int \psi_{at\mu}(r_{at} \approx 0, r_\mu)\phi(r_\mu, r_\alpha)\chi(r_n)d^3\underline{r}_\mu d^3\underline{r}_\alpha d^3\underline{r}_n \tag{2}$$

where $\underline{r}_\mu$, $\underline{r}_\alpha$, $\underline{r}_n$ represent the radial vectors of the subscripted particles, $\psi_{at\mu}(r_{at} \approx 0, r_\mu)$ is the initial wavefunction at nuclear contact, $\phi\ (r_\mu, r_\alpha)$ is the combined μ-α wavefunction including their correlation due to their coulomb attraction. $\chi(r_n)$ refers to the final state neutron and G is the relevant reaction constant.

The phase space constraints arising from the delta function conditions are insensitive to the details of the matrix element, as these are involved in the final energy integral to obtain the process rate. Introducing phase space factors, the Lorentz covariant rate for eq. (1a) can be written as

$$\Gamma = \int |M|^2 B\delta^4(P_i - P_\alpha - P_\mu - P_n)d^3\underline{p}_\alpha d^3\underline{p}_n d^3\underline{p}_\mu \tag{3}$$

where B includes the constants and the normalisation factors. P_i corresponds to total initial four momentum and P_α, P_n, P_μ to the four momenta of the final particles and $\underline{p}_\alpha$, $\underline{p}_n$, $\underline{p}_\mu$ are the respective momenta.

Integrating over the muon momentum delta function, Γ reduces to

$$\Gamma = \int |M|^2 B d^3\underline{P}_\alpha d^3\underline{P}_n \delta^0(E_i - E_\alpha - E_n - E_\mu^0) \tag{4}$$

where E_i, E_α, E_n referto energies and E_μ^0 is now fixed by the momentum conservation used in the lab-frame where the initial $(dt\mu)$ muo-molecule at rest i.e.

$$\underline{p}_\mu = -(\underline{p}_n + \underline{p}_\alpha) \tag{5}$$

So,

$$(E_\mu^0)^2 = E_\alpha^2 + E_n^2 - m_\alpha^2 - m_n^2 + 2p_\alpha p_n u + m_\mu^2 \tag{6}$$

where $p_\alpha = |\underline{p}_\alpha|$, $p_n = |\underline{p}_n|$ and u is the cosine, of the angle between $\underline{p}_\alpha$ and $\underline{p}_n$.

Integrating over the two ϕ angles and over the angle of the vector $\underline{p}_\alpha$, and going over the variables E_α, E_n

$$\Gamma = \int |M| \, B' E_n dE_n du E_\alpha dE_\alpha p_\alpha p_n \delta^0 (E_i - E_\alpha - E_n - E_\mu^0) \tag{7}$$

where B' absorbs the factors coming from the angular integration.

We integrate over E_α with the energy delta function (Bjorken and Drell 1964). Thus writing $f(E_\alpha) = E_i - E_\alpha - E_n - E_\mu^0$,

$$\Gamma = \int \frac{|M|^2 B' p_n E_n dE_n du p_\alpha^0 E_\alpha^0}{\left[\frac{df(E_\alpha)}{dE_\alpha}\right]_{E_\alpha = E}} \tag{8}$$

where E_α^0 satisfies $f(E_\alpha) = 0$ to coincide with the energy delta function constraint. The equation for E_α^0 using eq. (6) becomes

$$a(E_\alpha^0)^2 + b(E_\alpha^0) + c = 0 \tag{9}$$

where

$$a = (E_i - E_n)^2 - p_n^2 u^2,$$

$$b = -(E_i - E_n)[E_i^2 - \beta - 2E_i E_n]$$

$$c = \tfrac{1}{4}[E_i^2 - \beta - 2E_i E_n] + m_\alpha^2 p_n^2 u^2$$

and

$$\beta = m_\mu^2 - m_\alpha^2 - m_n^2$$

having solutions as

$$E_\alpha^0 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{10}$$

The quantity $g = (b^2 - 4ac)$ was computed for different values of E_n and u.

As positive value of g ensures the reality of E_α^0, and $E_\alpha^0 \geqslant m_\alpha$ ensures the positivity of the kinetic energy the resulting constraints on u for different values of of E_n were obtained. The kinematically allowed conical region and the forbidden zone are shown for a typical value of the neutron kinetic energy, T_n (Figure 1).

It is interesting to note that forbidden zone increases with T_n. On the other hand the constraint disappears for low neutron kinetic energies ; so that the whole domain of *u* is allowed. Figure 2 shows the variation of angle of the forbidden zone with kinetic energy of the neutron.

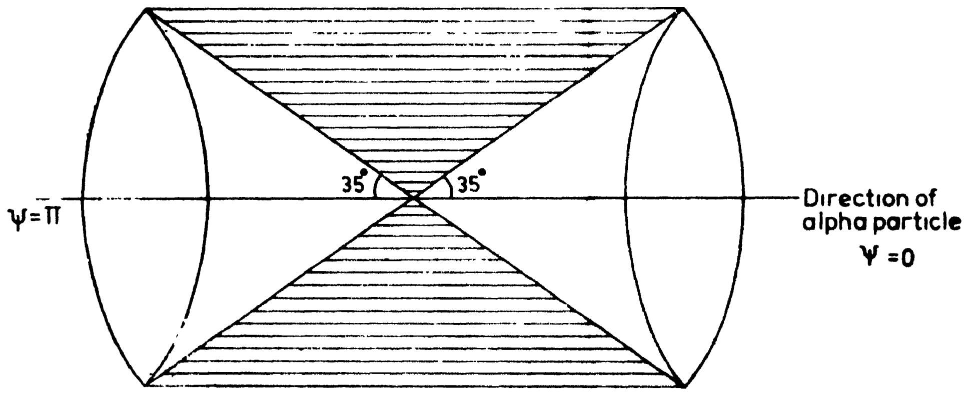

Figure 1. The forbidden zone (with lines) and the allowed cones for $T_n - 4.43$ MeV.

The direct sticking experiments (Paciotti *et al* 1988, Davis *et al* 1988) count alpha and neutron signals in coincidence at fixed configuration requiring alpha and neutron to be emitted in opposite direction i.e. angle π to each other.

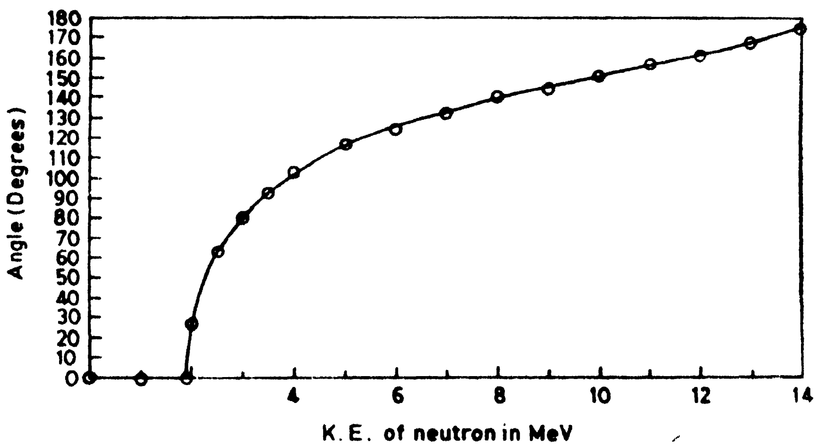

Figure 2. Variation of angles of forbidden zone with K. E. of neutron.

If one measures the energy of neutron and varies the inclination ψ of the neutron detector to the alpha detector the forbidden zone can be experimentally observed. The π angle corresponds to the static muon with alpha and neutron in opposite directions or the muon moving parallel to the alpha in a condition analogous to sticking.

It is therefore, encouraging that the phase space constraints predicted for the double differential rate for the muon catalysed fusion are detectable experimentally. These phase space conditions on the non-stuck fusion modes will also be reflected in estimates of the branching ratio for sticking as these depend on the non-stuck ratio. Correct collection of all non-stuck events by including the angular effects discussed in this paper, should serve to reduce the crucial sticking parameter and aid ultimate utilisation of cold fusion catalysed by muons.

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